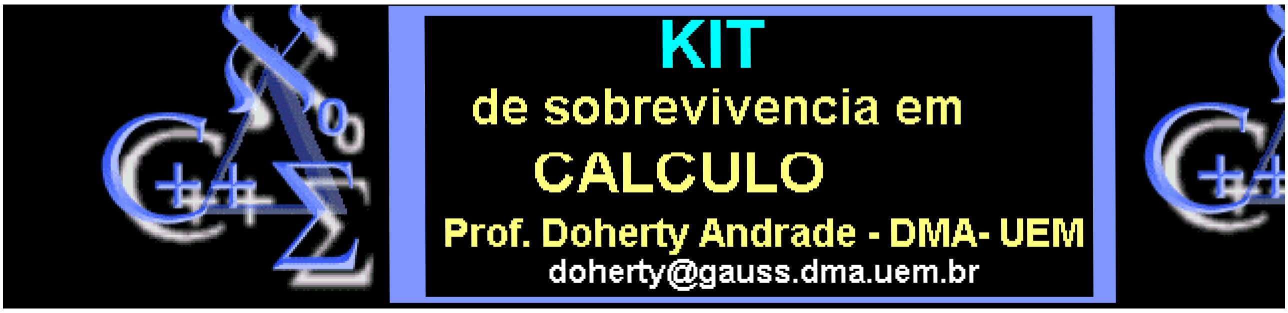

As aulas de 1 a 5 foram elaboradas juntamente com o Prof. Ma To FU (UEM)

NOÇÕES BÁSICAS

Álgebra Linear

Para acessar as funções relativas a Álgebra Linear, devemos

carregar a Biblioteca de Funções "pacote" linalg.

```
> #
> with(linalg):
```

Para se saber mais sobre qualquer uma da funções acima

listada, pode-se consultar o help iterativo. Por exemplo,

para se saber mais sobre "charmat": executa-se ?charmat;

Você pode um, dois ou três sinais de ?. O que cada um deles te dá?

```
> #
```

1 - DEFINIDO VETORES E MATRIZES

(Usando "vector", "matrix" e "array")

```
> u:=vector( [2,sin(x),4] );
```

$$u := [2, \sin(x), 4]$$

```
> v:=array( [ [1,1-sin(x),x] ] );
```

$$v := [1 \quad 1 - \sin(x) \quad x]$$

```
> vv:=convert(v, vector);
```

$$vv := [1, 1 - \sin(x), x]$$

```
> ut:=transpose(u); # tranposta de um vetor?
```

$$ut := \mathrm{transpose}(u)$$

```
> wt:=transpose(v); # transposta de um array
```

$$wt := \begin{bmatrix} 1 \\ 1 - \sin(x) \\ x \end{bmatrix}$$

```
> M:=matrix( [ [1, 2, -3], [x-3, 4, 0] , [2, 0 ,-1] ]);
```

$$M := \begin{bmatrix} 1 & 2 & -3 \\ x - 3 & 4 & 0 \\ 2 & 0 & -1 \end{bmatrix}$$

```
> N:=array( [ [1, 2, -3], [x-3, 4, 0] , [2, 0 ,-1] ] );
```

$$N := \begin{bmatrix} 1 & 2 & -3 \\ x - 3 & 4 & 0 \\ 2 & 0 & -1 \end{bmatrix}$$

```
> #
> # para adicionar vetores e matrizes
> # é: "+" e "evalm"
> #
> u+vv;
```

$$u + vv$$

```
> evalm(");
```

$$[3, 1, 4 + x]$$

```
> #
> MM:=matrix([[1,2,3],[0,1,-1],[0,0,1]]);
```

$$MM := \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

```
> NN:=evalm(2*MM);
```

$$NN := \begin{bmatrix} 2 & 4 & 6 \\ 0 & 2 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

```
> MM+NN;
```

$$MM + NN$$

```
> evalm(");
```

$$\begin{bmatrix} 3 & 6 & 9 \\ 0 & 3 & -3 \\ 0 & 0 & 3 \end{bmatrix}$$

```
> # Multiplicando Matrizes com "multiply" ou "&*"
> #
> C:=multiply(MM,NN);
```

$$C := \begin{bmatrix} 2 & 8 & 8 \\ 0 & 2 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

```
> F:=evalm(MM &* NN);
```

$$F := \begin{bmatrix} 2 & 8 & 8 \\ 0 & 2 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

```
> evalm(MM^3);
```

$$\begin{bmatrix} 1 & 6 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

Mais coisas básicas...

```
> M1:=array(1..3,1..3,[[a,b,c],[1,2,3],[alpha,beta,gamma]]);
```

$$M1 := \begin{bmatrix} a & b & c \\ 1 & 2 & 3 \\ \alpha & \beta & \gamma \end{bmatrix}$$

> **det(M1);**

$$2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c$$

> **M1_inversa:=inverse(M1);**

M1_inversa :=

$$\left[-\frac{-2\,\gamma+3\,\beta}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c},\frac{-b\,\gamma+c\,\beta}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-}\right.$$
$$\left.\frac{3\,b-2\,c}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c}\right]$$
$$\left[\frac{-\gamma+3\,\alpha}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c},-\frac{-a\,\gamma+\alpha\,c}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-}\right.$$
$$\left.-\frac{3\,a-c}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c}\right]$$
$$\left[-\frac{-\beta+2\,\alpha}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c},\frac{-a\,\beta+\alpha\,b}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-}\right.$$
$$\left.\frac{2\,a-b}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c}\right]$$

> **multiply(M1, M1_inversa);**

$$\left[-\frac{a\,(-2\,\gamma+3\,\beta)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c}+\frac{b\,(-\gamma+3\,\alpha)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b}\right.$$
$$-\frac{c\,(-\beta+2\,\alpha)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c},\frac{a\,(-b\,\gamma+c\,\beta)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-}$$
$$-\frac{b\,(-a\,\gamma+\alpha\,c)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c}+\frac{c\,(-a\,\beta+\alpha\,b)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-}$$
$$\frac{a\,(3\,b-2\,c)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c}-\frac{b\,(3\,a-c)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2}$$
$$\left.+\frac{c\,(2\,a-b)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c}\right]$$
$$\left[-\frac{-2\,\gamma+3\,\beta}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c}+2\,\frac{-\gamma+3\,\alpha}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha}\right.$$
$$-3\,\frac{-\beta+2\,\alpha}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c},\frac{-b\,\gamma+c\,\beta}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b}$$
$$-2\,\frac{-a\,\gamma+\alpha\,c}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c}+3\,\frac{-a\,\beta+\alpha\,b}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\ldots}$$
$$\frac{3\,b-2\,c}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c}-2\,\frac{3\,a-c}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-}$$
$$\left.+3\,\frac{2\,a-b}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c}\right]$$

$$
\left[-\frac{\alpha\,(-2\,\gamma+3\,\beta)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c}+\frac{\beta\,(-\gamma+3\,\alpha)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b}\right.
$$

$$
-\frac{\gamma\,(-\beta+2\,\alpha)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c},\ \frac{\alpha\,(-b\,\gamma+c\,\beta)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-}
$$

$$
-\frac{\beta\,(-a\,\gamma+\alpha\,c)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c}+\frac{\gamma\,(-a\,\beta+\alpha\,b)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-}
$$

$$
\frac{\alpha\,(3\,b-2\,c)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c}-\frac{\beta\,(3\,a-c)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2}
$$

$$
\left.+\frac{\gamma\,(2\,a-b)}{2\,a\,\gamma-3\,a\,\beta-b\,\gamma+c\,\beta+3\,\alpha\,b-2\,\alpha\,c}\right]
$$

```
> # Abaixo usamos matrix( m,n,[lista]).
> M2:=matrix( 3 ,3 ,[1,4,4,-3,7,0,0,2,7]);
```

$$
M2 := \begin{bmatrix} 1 & 4 & 4 \\ -3 & 7 & 0 \\ 0 & 2 & 7 \end{bmatrix}
$$

```
> # Escalonado M2
> gausselim(M2);
```

$$
\begin{bmatrix} 1 & 4 & 4 \\ 0 & 2 & 7 \\ 0 & 0 & \frac{-109}{2} \end{bmatrix}
$$

```
> # Aumentando M2 com a Identidade.
> Id:=diag(1,1,1);
```

$$
Id := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

```
> M3:=extend(M2, 0, 3, 0 );
```

$$
M3 := \begin{bmatrix} 1 & 4 & 4 & 0 & 0 & 0 \\ -3 & 7 & 0 & 0 & 0 & 0 \\ 0 & 2 & 7 & 0 & 0 & 0 \end{bmatrix}
$$

```
> MA:=copyinto( Id, M3 ,1, 4 );
```

$$MA := \begin{bmatrix} 1 & 4 & 4 & 1 & 0 & 0 \\ -3 & 7 & 0 & 0 & 1 & 0 \\ 0 & 2 & 7 & 0 & 0 & 1 \end{bmatrix}$$

```
> gaussjord(MA);
```

$$\begin{bmatrix} 1 & 0 & 0 & \frac{49}{109} & \frac{-20}{109} & \frac{-28}{109} \\ 0 & 1 & 0 & \frac{21}{109} & \frac{7}{109} & \frac{-12}{109} \\ 0 & 0 & 1 & \frac{-6}{109} & \frac{-2}{109} & \frac{19}{109} \end{bmatrix}$$

```
> #
> # Resolvendo sistemas com "linsolve"
> #
> F:=array( [ [-1,2,4], [3 ,2 ,1], [6 ,0 ,-3] ] );
```

$$F := \begin{bmatrix} -1 & 2 & 4 \\ 3 & 2 & 1 \\ 6 & 0 & -3 \end{bmatrix}$$

```
> B:=array( [ [1], [2], [3] ] );
```

$$B := \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

```
> X:=linsolve(F,B);
```

$$X := \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

```
> # Testando a solução
> multiply(F,X);
```

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

>

Polinômio Característico, autovalores, etc...

>

> **C:=matrix([[2,5],[4,8]]);**

$$C := \begin{bmatrix} 2 & 5 \\ 4 & 8 \end{bmatrix}$$

> **p:=charpoly(C, lambda); # Polinomio característico**

$$p := \lambda^2 - 10\,\lambda - 4$$

> **solve(p, lambda);**

$$5 + \sqrt{29},\, 5 - \sqrt{29}$$

> **eigenvals(C);**

$$5 + \sqrt{29},\, 5 - \sqrt{29}$$

> **eigenvects(C, radical);**

$$\left[5 + \sqrt{29},\, 1,\, \left\{\left[1,\, \frac{3}{5} + \frac{1}{5}\sqrt{29}\right]\right\}\right],\, \left[5 - \sqrt{29},\, 1,\, \left\{\left[1,\, \frac{3}{5} - \frac{1}{5}\sqrt{29}\right]\right\}\right]$$

> **#**

Voltando a coisas básica

> **#**

> **vandermonde([1,0,2]);**

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & 4 \end{bmatrix}$$

> **VV:=vandermonde([x,y,z]);**

$$VV := \begin{bmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{bmatrix}$$

> det(VV);

$$y z^2 - y^2 z - x z^2 + x^2 z + x y^2 - x^2 y$$

> factor(");

$$-(-y + x)(z - y)(z - x)$$

Portanto se x,y,z,... são todos distintos, a matriz de vandermonde é sempre inversível.

> vandermonde([1,2,3]);

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}$$

> J:=matrix([[1,1,1],[1,2,4], [1,3,9]]);

$$J := \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}$$

> jordan(J);

$$\left[(35 + I\sqrt{106})^{1/3} + \frac{11}{(35 + I\sqrt{106})^{1/3}} + 4 , 0 , 0 \right]$$

$$\left[0 , -\frac{1}{2}(35 + I\sqrt{106})^{1/3} - \frac{11}{2}\frac{1}{(35 + I\sqrt{106})^{1/3}} + 4 + \frac{1}{2} I\sqrt{3}\left((35 + I\sqrt{106})^{1/3} - \frac{11}{(35 + I\sqrt{106})^{1/3}} \right), 0 \right]$$

$$\left[0 , 0 , -\frac{1}{2}(35 + I\sqrt{106})^{1/3} - \frac{11}{2}\frac{1}{(35 + I\sqrt{106})^{1/3}} + 4 - \frac{1}{2} I\sqrt{3}\left((35 + I\sqrt{106})^{1/3} - \frac{11}{(35 + I\sqrt{106})^{1/3}} \right) \right]$$

> **evalf(");**

$$\begin{bmatrix} 10.60311024 & 0 & 0 \\ 0 & .1514518720 & 0 \\ 0 & 0 & 1.245437886 \end{bmatrix}$$

>